Syllabus of PhD Preliminary Examination in Probability and Statistics

Probability and Statistics Prelim Committee
Department of Mathematics
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The PhD Preliminary Examination in Probability and Statistics is intended to determine whether a student has adequate knowledge at the undergraduate level in statistics and probability (including stochastic processes) to begin a Ph.D. thesis research in applied mathematics. The exam is written and graded by a committee of professors in the Department of Mathematics with expertise in probability and statistics.

The written exam consists of approximately 6 to 8 questions. The students are given four hours to take the exam. Students are not allowed use of notes or books, but allowed use of a calculator. A passing score is 70% or higher. Students who fail the exam can take the exam one more time.

Students interested in taking the exam should have taken the courses, Math 483 (Mathematical Statistics) and Math 486 (Stochastic Processes), or their equivalents. A student should prepare for the exam by studying the relevant material from several books listed at the end of this syllabus, and by reviewing problems of the practice test and of previously given exams.

Statistics

- Families of distributions: location and scale families
- Populations, samples, statistics, order statistics
- Sampling distributions in the normal case (t, chi-squared, F)
- Parametric point estimation: maximum likelihood, method of moments, Bayes estimation
- MLE and Fisher information, univariate and multivariate, Delta method
- Hypothesis testing: likelihood ratio tests, size, level, power, chi-square tests
- Interval estimation, bootstrap (?)
- Bayesian inference

- Simple linear regression model: estimation, hypothesis testing, confidence estimation

- Regression diagnostics: residuals, outliers, influence

- Variance stabilizing transforms and weighted least squares

- Regression on functions of several variables: estimation, hypothesis testing, confidence estimation, special design matrices (polynomial, qualitative predictors), diagnostics

- Analysis of variance models

Probability and Stochastic Processes

- Basic setup of probability theory (including sample spaces, conditional probability, independence).

- Discrete and continuous random variables, the basic distributions, sums of random variables, transformations of random variables

- Moment generating functions

- Convergence of random variables (convergence in distribution, probability, mean-square).

- Conditioning

- Law of large numbers, central limit theorem

- Discrete-time Markov chains, convergence, stationary distributions, first-step analysis

- Poisson process and continuous-time Markov Chains

References

Main

1. L. Wasserman, *All of Statistics: A Concise Course in Statistical Inference*, Springer, 2010.

2. R. Hogg, J. McKean, A. Craig, *Introduction to Mathematical Statistics*, Pearson, 2012.

3. M. Kutner, C. Nachtsheim, J. Neter and W. Li.*Applied Linear Statistical Models*, McGraw-Hill/Irwin, 2004

4. H. Taylor and S. Karlin, *An Introduction to Stochastic Modeling*, Academic Press, 2014.

Supplementary

1. (?) R. Walpole et al *Probability and Statistics for Engineers and Scientists*, Pearson, 2016.

2. L. Bain and M. Engelhardt, *Introduction to Probability and Mathematical Statistics* , Cengage, 2000.